Mathematical etude: killing two birds with one stone

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Mathematical etude: killing two birds with one stone

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Abstract. Procedural fluency is one of the essential capabilities that students need to possess in mathematics. To put it simply, procedural fluency is the knowledge of how to conduct a mathematical procedure. Repetitive practice or drill are usually employed to improve students' procedural knowledge. On the other hand, many others say that mathematics learning should set a higher goal, rather than just 'knowing how to do'. Instead of drilling, open-ended tasks are said to be the best match and to improve problem solving skills and creativity. Mathematical etude offers both of them. This study aimed to help students develop deeper insights of rich-tasks like Mathematical Etude. A total of 28 middle school students were administered the Mathematical Etude task sheet and were interviewed about their obstacles in solving the task. The data obtained were qualitatively analysed. The results showed that most students found the Mathematical Etude was challenging because they have not got used to such tasks. More findings and suggestions were elaborated in the text.

1. Introduction

Mathematical proficiency is like ropes consisted of intertwined strands. The strands are not independent and represent different aspects which empowers each other. Procedural fluency is one of the mathematical proficiency that students should have to be able to learn mathematics successfully [1]. According to Kilpatrick et al.[1], procedural fluency or procedural knowledge is the knowledge about procedures as well as when and how to use them appropriately, flexibly, accurately, and efficiently. The role of procedural knowledge in mathematics has become a discussion topic in mathematics education community. Some said that its role is just as a compliment to the conceptual knowledge [2]. Another said mathematics learning with procedural fluency as the goal will be only about rote, not meaningful understanding and will have a negative impact on students.

However, Star [3] stated that the preceding disagreements might be because of the lack of research on procedural knowledge and its development. Star [3] added that there are superficial and deep procedural knowledge. Someone with superficial procedural fluency only uses a standard technique and possibly is not able to solve unfamiliar problems. While deep procedural fluency refers to the ability to navigate the procedures, use other techniques rather than the overpracticed ones, and produce solutions that best fit the problem conditions.

What comes next is not to consider procedural-based instruction as inherently harmful. In fact, the automation of procedure may help students solve mathematics problems in wider contexts. The teachers' role is to ensure that students deal with the procedures in a critical way so that students can regard them as useful tools to solve mathematical problems [4].

Dealing with procedures in a critical way will not be accomplished by using repetitive practice or drill. Teachers need to provide students with rich tasks which allow students to 'get inside' mathematics [5]. The rich tasks can offer opportunities to deepen the ability to solve problems with correct procedures and in the same time encourage creativity of students.

Foster [6] started the Mathematial Etude Project as the answer for the double demands of mathematics proficiency. The term Etude which is commonly used in the music field is a French word for 'study'. According to Britannica [7], Etude is originally a study or technical exercise, later a complete and musically intelligible composition exploring a particular technical problem in an esthetically satisfying manner. Like the name, Etude was originally used as a didactic piece or practice repertoire. In the late 18th centuries, etude started to be performed in a concert for entertainment purposes. The double objectives of the musical etude is what came accross Foster's mind and inspired him to introduce the notion of Mathematical Etude [8]. The current study described how this task was solved by Indonesian students as well as the challenges and strategies used by them.

2. Research Method

This research is descriptive-exploratory research aimed at exploring students' experiences in solving a Mathematical Etude problem. Mathematical etude is an accessible worksheet on one specific topic in mathematics. A total of 28 students were interviewed related to the translated version of the Mathematical Etude task sheet that they had completed. The Polygons Expression [9] task sheet was employed considering the algebraic expression as fundamental concept in secondary school.

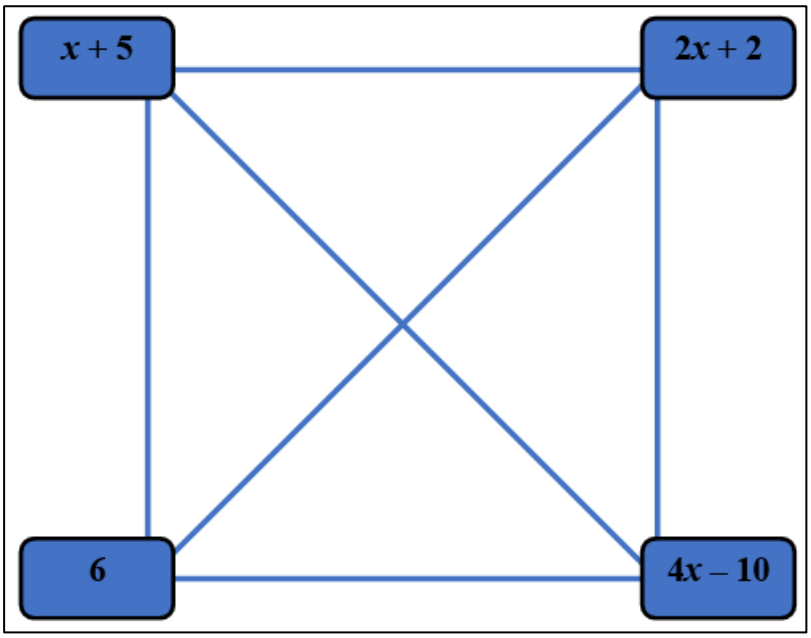

Figure 1. Algebraic Expression Polygon

Students then were about to answer four questions, namely:

(1) Write the six equations represented by the six line segments in this expression square.
(2) Solve those six equations.
(3) Describe any patterns you notice among your six solutions.
(4) Construct another polygon expression containing different expressions. Can you make the solutions to your expression polygon a "nice" set of numbers?

3. Results and Discussion

The task consists of two parts. The first two questions solely assessed students' procedural fluency in solving linear equations. Students' answers can lead to insight into their strategies and errors. Participants were expected to answer these questions correctly since they are 9th graders while solving one-variable linear equations were taught in 7th grade. However, the findings reported the opposite. It was found that although some students answered correctly (Figure 2) more than half of the participants answered the second questions incorrectly. Figure 3 showed students' errors related to the positive and negative sign (3a) and computation-related mistakes (3b).

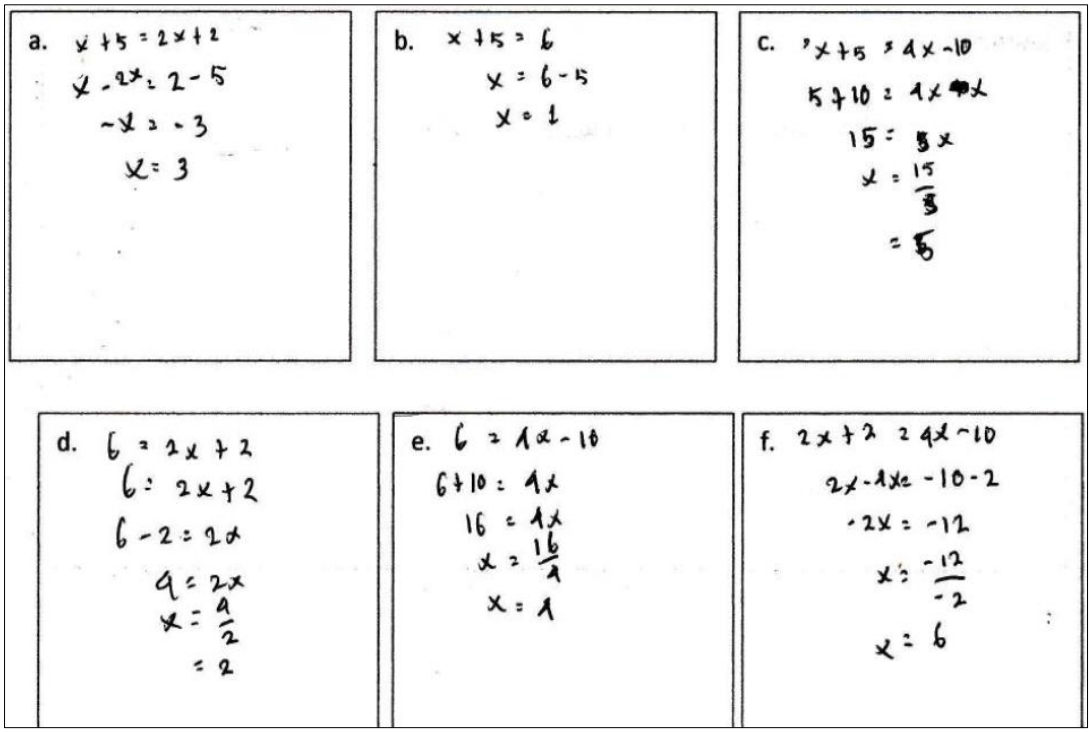

Figure 2. Sample of Students' Correct Answer

The other two questions required students to know more than just how to solve linear equations. The third question asked students to identify the patterns formed by the solutions. The answer is 1, 2, 3, 4, 5, and 6 or the first six natural numbers. The students who got the second question wrong obviously would answer the third question wrong.

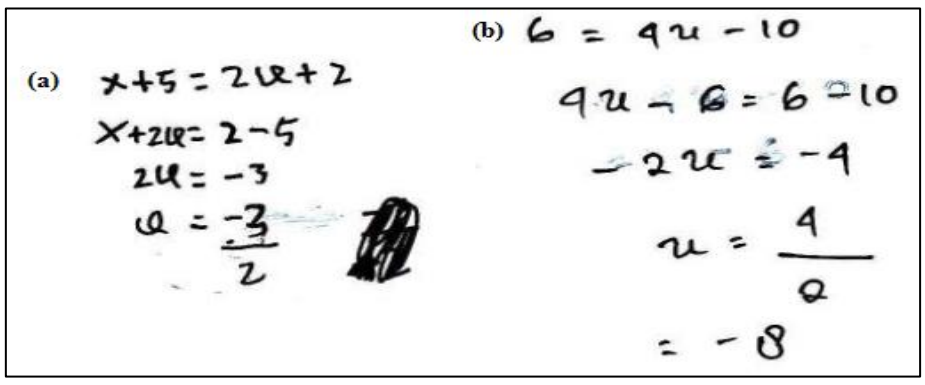

Figure 3. Students' Error Regarding: (a) Positive and Negative Signs, (b) Computation

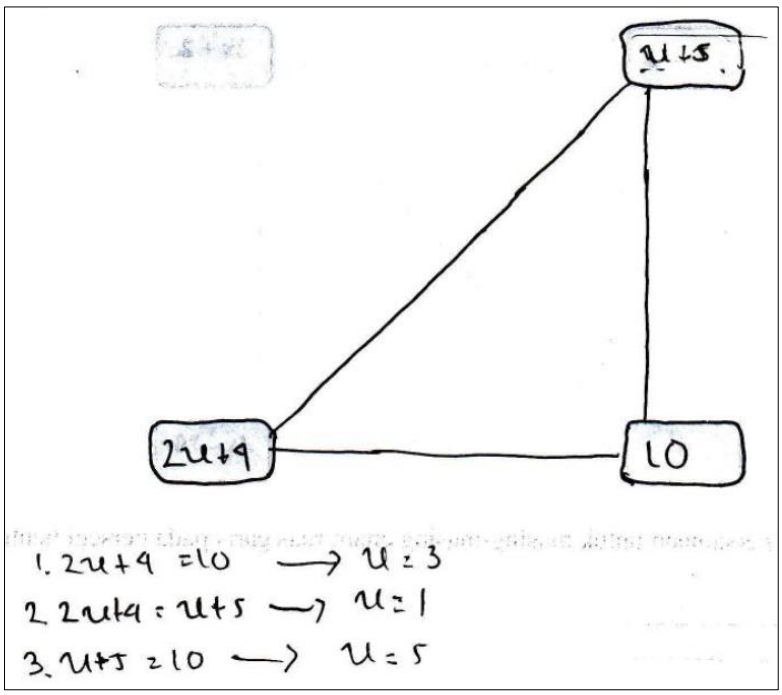

Figure 4. The Correct Answer for the Last Question

The fourth question is the divergent task where the students were given opportunities to improve their creativity and problem solving skills. They were asked to define their own pattern of numbers and arrange their own expression polygon using the predefined set of numbers. This part is said to be provocative yet challenging for students [10]. As for the current study, it is indeed a challenging task since the students were not familiar with this kind of task [11]. Statements like "I feel that the second questions is the easiest since I already knew how to solve it" and "I encounter this kind of questions (number 2) the most, so I can solve it easily" were the most common response from students. After seeing the students struggle in finding four algebraic expressions, teachers then lowered the level by using a triangle, instead of square like the first question. Nevertheless, lowering the difficulty level did not bring significant change. Out of 28 participants, there is only one person who could make an expression polygon with a 'neat' solution (Figure 4). The participant decided to make an expression triangle which requires three algebraic expressions and three solutions, namely 1, 3 and 5 (the first three odd numbers). "After I decided to use 1, 3, and 5, all I did was trial and error until I found the three equations" he stated.

4. Conclusion

Repetitive practice or drill should not be an only option for teacher to use in the mathematics classroom anymore. Procedural knowledge or procedural ability is important but we have to ensure the students working on the task where they can build broader and deeper creativity and problem solving skills, instead of having the same meaningless practice all over again. Mathematical etude could be an alternative for teachers. However, since this is for practice purposes and not for summative assessment, students need to work on the worksheets under teacher's guidance and assistance. If the objective of mathematical instruction is far more than just 'knowledge how to do' then teachers need to convert from the repetitive practice to rich-tasks like Mathematical Etude. Further study should be conducted to investigate the implementation and influence of rich task like Mathematical Etude.

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